"Equation" means "equal"

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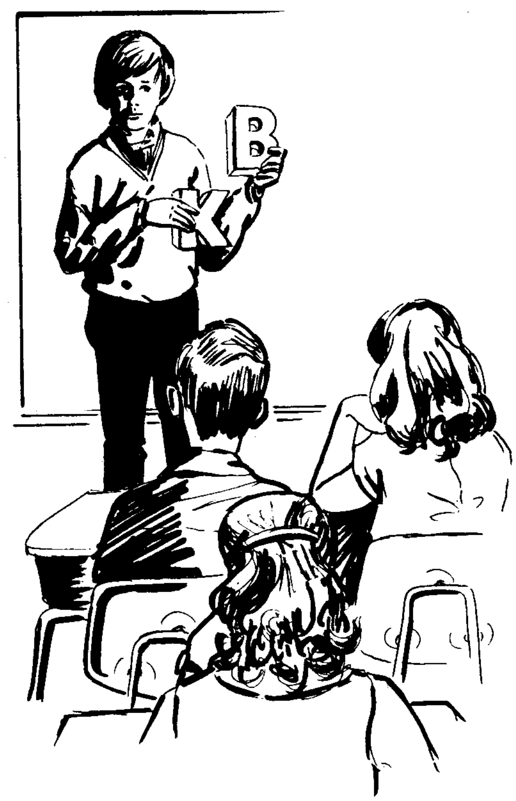
Having recently received her Ph.D. at the University of Maryland, Barbara Bernstein now teaches mathematics full-time at Bowie State College in Maryland. She has developed teaching techniques like the one described here through frequent tutoring of students having difficulty in mathematics.

In late elementary school, students are introduced to translating sentences into simple algebraic equations. For example, students may be asked to express the following in mathematical symbols: Bill has three more model planes than Ken. Many students find it difficult to distinguish whether $B + 3 = K$ or $B = K + 3$ is correct (where B equals the number of planes Bill has and K equals the number of planes Ken has). I have found a procedure that helps students over this common stumbling block.



The student imagines that there are two vertical tape measures extending from the floor to the ceiling, with the floor having a height of zero units. The student uses the vertical position of one hand to represent the number of Bill's planes (B) and the vertical position of the other hand to represent the number of Ken's planes (K). Elementary school students generally have enough English language reading ability to realize that the statement "Bill has three more planes than Ken" means that Bill is the fellow with more planes. So once the student has let the level of one hand represent B, he sees that the hand representing K will be lower.

But how much lower is the second hand? The statement tells us it is three units lower; this brings us to the writing of an equation. Often students do not fully understand that an equation is a statement of equality. It may be useful to point out that *equation* practically has the word *equal* in it. The teacher can explain as follows: Since B and K are not equal, some operation must be done to one of the variables to get it equal to the other one before an equation can be written.

The teacher can then ask the students, "What can be done to make these two hands be at the same level?" With the hands right before the children's eyes it is obvious that, as I like to put it, "You either have to add three to the number of Ken's planes to bring it up to the level of Bill's planes, or you must subtract three from the number of Bill's planes to bring it down to the level of Ken's planes."

In the first case, since $K + 3$ is then at the same vertical level as B, we can write $K + 3 = B$. Or in the latter case, $B - 3$ is down at the level of K so we have $K = B - 3$. (Since students often make the error of writing $3 - B$, it should be stressed that one must start with the quantity B and subtract 3 *from* it.) Note that subtracting 3 from both sides of $K + 3 = B$ yields $K = B - 3$. This can be pointed out to the students to demonstrate the meaning and validity of using the additive inverse to solve equations.

If much discussion ensues between steps while the teacher is presenting this method, the students may forget the all-important detail of which hand represents which variable. A lively way to avoid confusion is to cut the letters of the variables out of colored paper; the student can then hold the appropriate one in each hand.

Students can also use this method to determine which variable is *multiplied* by a constant, if that is what the problem calls for. The procedure I have described helps a student visualize the problem and, as so many teachers have found, visualizing aids understanding.