# "The Advantages of Using Mathematics History in Math Education"

by
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It is curious that among all secondary school subjects, it is in mathematics that teachers make the least use of the history of the discipline. Typically, the only text reference to history is a one-page section at the end of some chapters, such as Dolciani's "Human Angle" article. These short notes on historical figures and developments are not meaningfully integrated into the material; doubtless most students skip over them and never achieve the sense of perspective which historical relatedness can offer.

Mathematics educators should take a lesson from science education which capitalizes on the history of scientific discoveries. In the BSCS series, for example, many chapters give ample coverage to the historical development of major concepts. Mathematics, on the other hand, is treated as a body of knowledge which is organized for maximally efficient assimilation by the student. The crude origins of those concepts are rarely mentioned; they are only presented in final, polished form.

Furthermore, much of mathematics remains magical to students for they do not understand the relationship it bears to the world. In recent years students have decried the lack of "relevance" in their education, and mathematics is often hardest hit by these complaints. Providing a sense of the history of mathematics is a natural way to supply the discipline with relevance while not sacrificing academic material. In the remainder of this paper I will consider a few common topics in secondary mathematics and suggest how historical notes could be used to improve teaching.

In junior high school when students are studying the properties of numbers, they would find the Pythagorean "numerical philosophy of life" quite interesting. The Pythagoreans believed that numbers have extraordinary attributes. "One" was thought to be the essence or source of all things. Numbers were divided into evens and odds, the former were regarded as feminine and the latter as masculine. Because "five" is the union of the first even and odd numbers, it was thought to signify marriage. "Six" was identified with cold, and "seven" with health and light. Two numbers were thought to be "friendly" if each was the sum of divisors of the other, such as 200 and 284. (Students might enjoy an assignment to find other pairs of "friendly" numbers.)

Though these speculations in themselves are fanciful and barren, fruitful lines of mathematical inquiry are suggested by them. The mystical underpinings of this early mathematics cannot fail to fascinate students and it would give them a sense of the crude origins out of which mathematics, like other disciplines, began. It is this sense of reality and worldliness which is grossly lacking in the efficiently organized body of mathematical knowledge we now teach.

The Pythagorean discovery of irrationals also deserves note for it influenced the development of mathematics for twenty centuries. The concept of irrationality is deep and a lesson on its historical discovery would considerably aid the students' understanding. The teacher should lead the class in thinking through the genesis of this idea.

The discovery is presumed to have grown out of the study of the isosceles right triangle. If the legs are set equal to "one" then no number can be found which exactly represents the length of the hypoteneuse in accord with the law $a^2 + b^2 = c^2$. Students should be encouraged to try squaring some fractions so they see for themselves the impossibility of finding an exact rational root of two.

The teacher could explain the dilemma by saying that it is reasonable to seek a number system in which there is some number that represents the exact length of any line that can be drawn. Yet use of the proven formula $a^2 + b^2 = c^2$ inevitably leads to the fact that there are some kinds of numbers which are not included in the set of rationals that the Greeks recognized.

The legend surrounding this discovery is amusing and it would impress upon the students how central these findings are to mathematics. The Pythagoreans saw something "unspeakable" in the irrationals and they all promised to keep the discovery a secret. The man who divulged the secret was said to have died in a shipwrect as a result. The concept of irrationality is easier to understand when its historical importance is recognized.

Another highlight of mathematics which should be pointed out in teaching is Descartes' and Fermat's discovery of the use of algebra for representing and analyzing curves. Toward the end of the sixteenth century there was increasing need to develop new mathematical tools to accommodate advances that were being made in other fields. The Copernican theory called for the use of ellipses, parabolas and hyperbolas in representing the motion of the heavenly bodies. The motion of pendula, objects suspended from springs, and paths of projectiles were also studied by the use of curves.

Meanwhile during the late sixteenth and early seventeenth centuries much progress was made in algebra. This culminated in the discovery of the algebraic representation of curves which enabled men to learn a lot about curves by working with their equations. Since the discovery is so basic to science, Descartes and Fermat can be considered the founders of mathematical physics.

The Cartesian coordinate system unified algebra and geometry and provided a tool for sophisticated mathematics. Yet we usually explain its use to students with no reference to the key role that this development played in history. The use of algebra and a coordinate system to represent points seems simple and natural. But we should explain to students that this technique was an important step in both mathematics and science. Moreover the kinds of physical problems which motivated the development of coordinate geometry (such as the path of projectiles) are things that youngsters have experience with and can appreciate.

Another area which deserves historical consideration in teaching is non-Euclidean geometry. By altering the parallel postulate, "bizarre" systems of geometry can be generated. For a long time these new geometries were regarded as logical curiosities. Yet in the theory of relativity, Einstein employed non-Euclidean geometry and the agreement between his predictions and observations was closer than it would have been had he used Euclidean geometry. So non-Euclidean properties hold when the space under consideration is our universe. Yet over the years Euclidean geometry has proven valid for the solution of earthly physical problems such as building bridges.

It is clear that Euclidean and non-Euclidean geometries are valid in different physical spaces. We use the one which best fits experience with respect to a particular problem, recognizing that none of these systems is true in an absolute sense. The development of non-Euclidean geometries forced men to realize that mathematics, like science, provides a theory about space which can be discarded when a better theory presents itself.

Non-Euclidean geometry has had a profound effect not only on mathematics but also on other branches of culture. Mathematical facts had been considered indisputable truths for years. The Greek and Newtonian view of things was that through mathematics and physical observations, man uncovers the design inherent in nature. But with the discovery that mathematics does not provide absolute truths, men realized that they can achieve at best only good approximations of nature. No matter how accurate, these formulations are nothing more than man's way of understanding nature. Indeed Morris Kline claims that the development of non-Euclidean geometry is one of the two ideas which most profoundly influenced Western intellectual thought since the nineteenth century, for the abandonment of absolutes has "seeped into the minds of all intellectuals."

The impact of these systems on modern thought would make a fascinating lecture for high school students. Students have said that math bears no relation to daily life. Yet the indirect effect of mathematics on daily life has been profound since the lack of absolute truths which pervades twentieth century thinking has its roots in mathematical and physical discoveries. Generally Euclidean geometry is taught only in high school and sometimes brief coverage is given to the basics of non-Euclidean geometry. But the significance of these systems is rarely mentioned though it would put the rest of the year's study of Euclidean geometry in perspective.

The topics I have discussed in this paper were keynotes in the development of mathematics. Not all topics in secondary math play such a central role in history. But every concept has some heritage which deserves our consideration in teaching. The suggestions I have outlined would have to be altered to suit the sophistication level of the class. This teaching tool has potential for both improving students' understanding of mathematical material and helping them to see how that material relates to the history of mathematics and other fields of thought.

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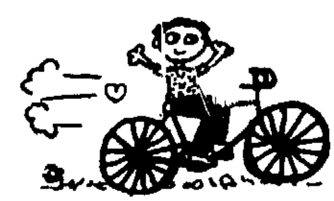

**"I ain't got no idea why I didn't pass English"!**